Red Cab, Blue Cab

You act for a pedestrian struck by a car in a hit and run accident. Your client cannot identify the car that hit him, but is sure that it was a taxicab. The only witness was an elderly woman. She has identified the car that struck your client as a Blue Cab.

There are 100 taxicabs in town. The Red Cab Company owns 85, all painted red. The Blue Cab Company owns the other 15, all painted blue.

You are concerned about the strength of your identification evidence. Therefore, before starting an action against the Blue Cab Company, you hire an expert to evaluate this witness's ability to distinguish between Red and Blue Cabs. Your expert runs a series of tests and concludes that under the conditions prevailing at the accident scene, this woman correctly distinguishes between Red Cabs and Blue Cabs 80% of the time. You are satisfied that the expert evidence is unassailable.

Accepting that your witness correctly distinguishes Red Cabs from Blue Cabs 80% of the time, what is the probability that it actually was a Blue Cab that struck your client?

Choose one answer from A, B, or C.

A. I am comfortable that the answer is _______ %.

B. I am not comfortable giving a precise figure, but I am comfortable that the probability is in the range of (choose one):

99% - 80% **79% - 60%** **59% - 40%** **39% - 20%** **20% - 0%**

C. I am not comfortable giving a precise figure or picking a range, but I am comfortable in saying that, on the balance of probability, it was a Blue Cab that struck my client.

Yes **No**

Testing, Testing

The School Bus Driver Company provides school bus drivers for elementary schools and employs several thousand drivers.

The company President has just learned that the rate of intravenous opiate use in the general population is roughly 1 user for every 1000 adults. He is advised by experts that he can expect the rate of intravenous opiate use for his company's employees to be the same as it is in the general population. Concerned, he decides to introduce a drug testing program.

A reputable testing company is hired to operate the program, using a state-of-the-art test that always returns a positive result for anyone who actually is an intravenous opiate user (*i.e.*, its negative results are 100% accurate), but also has a false positive rate of 1%.

What is the probability that an employee who shows a positive result from this test actually is an intravenous opiate user?

Choose one answer from A, B, or C.

A. I am comfortable that the answer is _______ %.

B. I am not comfortable giving a precise figure, but I am comfortable that the probability is in the range of (choose one):

99% - 80% **79% - 60%** **59% - 40%** **39% - 20%** **20% - 0%**

C. I am not comfortable giving a precise figure or picking a range, but I am comfortable that in saying that, on the balance of probability, an employee who has a positive result on this test is an intravenous opiate user.

Yes **No**

The Monty Hall Problem

Monty Hall was the best known host of the long-running U.S. television game show “Let’s Make A Deal!” At the end of every show, Monty would offer the leading prize winner from that day’s show a chance to risk the day’s winnings for a chance to win a Grand Prize.

The contestant would be shown three closed doors, and given the option to choose one of them to be opened. Behind one of the doors would be a collection of valuable and desirable items (*e.g*., cars, expensive vacations, high-end furniture suites). Behind the other two doors, there would be no prize at all, or a joke consolation prize (*e.g*., a lawn-trimming goat, a bicycle-powered television).

A contestant that decided to take this final gamble would select one door to be opened. However, before he opened that door, Monty would open one of the two unselected doors to reveal what was behind it. Because Monty knew which door concealed the Grand Prize, the door he chose to open always revealed either the joke consolation prize or nothing at all.

The contestant was then given a choice between keeping the door originally selected, or switching that selection for the remaining closed door.

Which naturally gave rise to the question “Does one strategy (switch or keep) give you better odds of winning the Grand Prize, or are your odds of winning the same, no matter what you do (switch or keep)?”

Choose one answer from A, B, or C. For answers B & C, giving the percentages is optional.

A. It makes no difference whether you keep your original selection or switch. Either way, your odds of winning are 50%.

B. You should always keep your original selection. [This way, your odds of winning are ______ %, as opposed to ______ % if you switch.]

C. You should always switch. [This way, your odds of winning are ______ %, as opposed to ______ % if you keep your original selection.]

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A reputable testing company is hired to operate the program, using a state-of-the-art test that always returns a positive result for anyone who actually is an intravenous opiate user (*i.e.*, its negative results are 100% accurate), but also has a false positive rate of 1%.

What is the probability that an employee who shows a positive result from this test actually is an intravenous opiate user?

The incidence of intravenous opiate drug use in the general population is 1 in 1000, so we can expect that 1 out of every 1000 School Bus Driver Company employees will actually be an intravenous opiate drug user.

When those 1000 employees are tested, because the test never fails to detect actual drug users, that 1 user will deliver a true positive test result.

However, because the test has a false positive rate of 1% (which is quite a low false positive rate for most drug tests), 10 employees who are not intravenous opiate drug users will generate false positives. In total, then, the test will deliver 11 positive results for every 1,000 employees tested:— 1 will be a true positive, 10 will be false positives.

Therefore, the odds are roughly 11 to 1 against a positive test result actually indicating opiate use. In other words, there is only a 9% probability that an employee who tests positive is actually an opiate user. Conversely, it is more than 90% certain that an employee who tests positive is NOT an intravenous opiate drug user.

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Accepting that your witness correctly distinguishes Red Cabs from Blue Cabs 80% of the time, what is the probability that it actually was a Blue Cab that struck your client?

An easy way to arrive at the correct probability is to ask what the ratio of correct to incorrect Blue Cab identifications would be if the witness saw all 100 taxis in town strike your client.

There are 15 Blue Cabs in town. When seeing those 15 taxicabs strike your client, the witness would correctly identify the car as a Blue Cab 80% of the time, but incorrectly identify it as a Red Cab 20% of the time. She would therefore report seeing 12 Blue Cabs (80% of 15) and 3 Red Cabs (20% of 15) strike your client.

There are 85 Red Cabs in town. When seeing those 85 taxicabs strike your client, the witness would correctly identify the car as a Red Cab 80% of the time, but incorrectly identify it as a Blue Cab 20% of the time. She would therefore report seeing 68 Red Cabs (80% of 85) and 17 Blue Cabs strike your client.

After seeing all 100 collisions, the witness would therefore report that your client was struck by 71 Red Cabs (3+68) and 29 Blue Cabs (12+17). Of the witness's 29 reports identifying the vehicle as a Blue Cab, only 12 would be accurate. Thus, the probability that, in this instance, a Blue Cab actually struck your client is 12/29, or 41%.

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The Monty Hall problem actually became a serious embarrassment for the academic mathematical establishment in the United States. A newspaper columnist, Marilyn vos Savant, published a column in which she concluded that it always made sense to switch. She was not a professional mathematician, and her original analysis of the problem wasn't very elegant, and it attracted a stream of derisive comments from the public at large, as well as from many prominent academic mathematicians, all of whom thought it obvious that it made no difference whether you switched or not. Indeed, many of the academic commentators (including one Department Chair) used the occasion of this benighted woman's elementary error (as they saw it) to launch Jeremiads on the lamentable state of U.S. math education.

The proper objects for their scorn were, of course, themselves. Ms. Savant was quite right. You should always switch. It improves your odds of winning by about 50% (from 1/3 to 1/2). Here's why that is so.

When you make your original choice, there are 3 closed doors, 1 of which conceals the Grand Prize. Therefore, *when you make that selection*, your odds of picking the winning door are 1 in 3, or 33%. The odds of success for this original selection are created by the original configuration (1 in 3) and therefore do not change as a result of subsequent events.

After Monty opens one of the other doors, you are left with two closed doors, one of which conceals the Grand Prize. In this new configuration, the chance that the unselected closed door conceals the Grand Prize is 1 in 2, or 50%. By switching to that door, you upgrade your 33% chance of having selected the winning door to a 50% chance of selecting the winning door.